Rugate absorbing thin films and the 2 × 2 inhomogeneous matrix

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The 2 × 2 matrix formalism proposed by Bovard is extended to absorbing inhomogeneous thin films for propagating the electromagnetic field. The contribution of each order into multiple reflections inside the film is analyzed.

Key words: Rugate filters, characteristic matrix.

The study of propagation of the electromagnetic field through dielectric inhomogeneous thin films has been considered in the past, and little research, to our knowledge, has been reported on inhomogeneous absorbing thin films.

Some scientists have applied the homogeneous matrix method to calculate the spectral performance of absorbing inhomogeneous thin films for solar absorbers.[1,2] In their analyses these coatings are hypothetically divided into many thin slices of constant refractive index to simulate the variable index profile.

A more powerful tool for studying these kinds of systems has been proposed by Bovard.[3] It involves the generalized matrix method for propagating the electromagnetic field through dielectric inhomogeneous thin films. In this context this study may be considered a natural extension of the inhomogeneous matrix to include absorbing rugates. This implies a detailed study of Maxwell's equations to obtain a matrix solution, and for simplicity we consider here the most important features of the generalization. We will see that the main contribution to the total spectral response is strongly located in the first orders of the multiple reflections, in contrast to dielectric materials, where the contributions are more uniform.

Let us consider an inhomogeneous absorbing thin film characterized by its complex index

$$N(z) = n(z) - ik(z), \tag{1}$$

where $n(z)$ and $k(z)$ are the refractive and absorption indices, respectively. Variable z represents the axis reference for propagation and the direction of the inhomogeneity. Assuming that we have a nonabsorbing incidence medium and following the procedure in Ref. 3 for solving Maxwell's equations, it is possible to obtain a matrix solution:

$$\begin{pmatrix} B \\ C \end{pmatrix} = \begin{Bmatrix} \left[\dfrac{\eta(\beta_0)}{\eta(0)}\right]^{1/2} L(\beta_0) & \dfrac{iG(\beta_0)}{[\eta(\beta_0)\eta(0)]^{1/2}} \\ i[\eta(\beta_0)\eta(0)]^{1/2} K(\beta_0) & \left[\dfrac{\eta(0)}{\eta(\beta_0)}\right]^{1/2} F(\beta_0) \end{Bmatrix} \begin{pmatrix} 1 \\ \eta_s \end{pmatrix}. \tag{2}$$

In contrast to the dielectric case the constants $[\eta(\beta_0)/\eta(0)]^{1/2}$ and $[\eta(\beta_0)\eta(0)]^{1/2}$ (given in terms of the admittance of the film on its boundaries) have two roots because $\eta(\beta)$ is complex. However, both roots give the same results when we are calculating the spectral performance. The angular dependence of the complex admittance of the film at a given point β is

$$\eta(\beta) = \begin{cases} \eta_0 \dfrac{N(\beta)}{\cos\hat{\theta}(\beta)}, & \text{TM case}, \\ \eta_0 N(\beta)\cos\hat{\theta}(\beta), & \text{TE case}, \end{cases} \tag{3}$$

where $\hat{\theta}(z)$ stands for the complex angle of propaga-

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tion and η_0 is the admittance of the free space. The phase integral $\beta(z)$ is

$$\beta(z) = \frac{2\pi}{\lambda} \int_0^z N(z) \cos \hat{\theta}(z) \mathrm{d}z, \tag{4}$$

where λ is the wavelength. In the matrix of Eq. (2), $B = E_i/E_s$ and $C = H_i/E_s$, the fields E_i, H_i represent the incident wave and E_s, H_s the transmitted field toward the substrate. The admittance of the substrate is denoted by η_s. The functions $F(\beta)$, $K(\beta)$, $G(\beta)$, and $L(\beta)$ are given in Ref. 3 in terms of a series of functions,

$$\hat{C}_m = \int_{\Gamma}\int_{\Gamma_1} \cdots \int_{\Gamma_{m-1}} r(\beta_1) r(\beta_2) \cdots r(\beta_m) \times \cos 2(\beta_m - \beta_{m-1} + \cdots \beta_1) \mathrm{d}\beta_m \cdots \mathrm{d}\beta_2 \mathrm{d}\beta_1, \tag{5}$$

$$\hat{S}_m = \int_{\Gamma}\int_{\Gamma_1} \cdots \int_{\Gamma_{m-1}} r(\beta_1) r(\beta_2) \cdots r(\beta_m) \times \sin 2(\beta_m - \beta_{m-1} + \cdots \beta_1) \mathrm{d}\beta_m \cdots \mathrm{d}\beta_2 \mathrm{d}\beta_1, \tag{6}$$

whose order is related to the number of internal reflections. Because β and $r(\beta)$ are complex functions when absorption is present, they are complex integrals of analytic functions that are evaluated along the paths $\Gamma, \Gamma_1, \ldots, \Gamma_{m-1}$ given by Eq. (4). The function of the complex index profile $r(\beta)$ is

$$r(\beta) = \frac{\eta'(\beta)}{2\eta(\beta)} \cdot \tag{7}$$

The matrix of Eq. (2) has a unity value determinant, and each of its elements corresponds to its counterpart in the well-known characteristic matrix for homogeneous films. Then spectral performance is expressed in terms of the equations for homogeneous filters.[4]

Expressions for reflectance, transmittance, and absorptance (R, T, A) can be simplified if the boundaries of the layer are extended inside the incident and

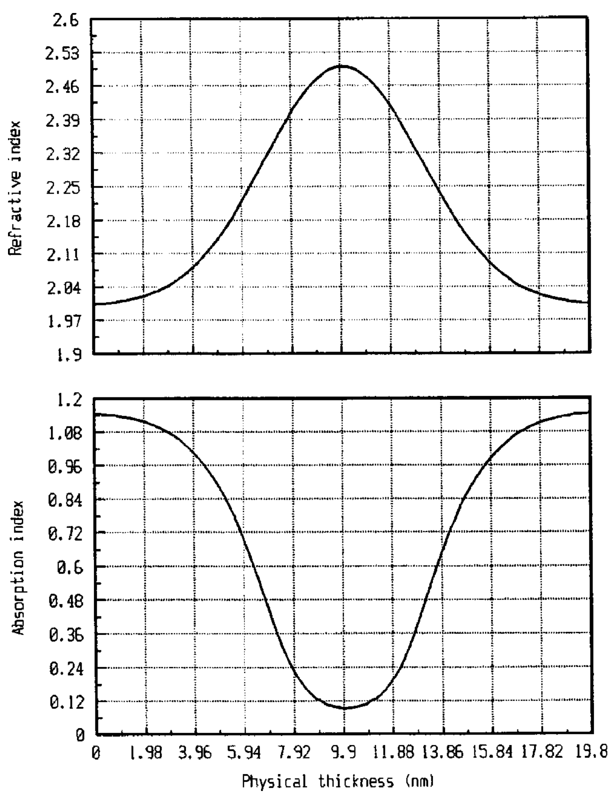

Fig. 1. Refractive- and absorption-index profiles of a beam splitter. The incident medium is air and substrate BK7 glass.

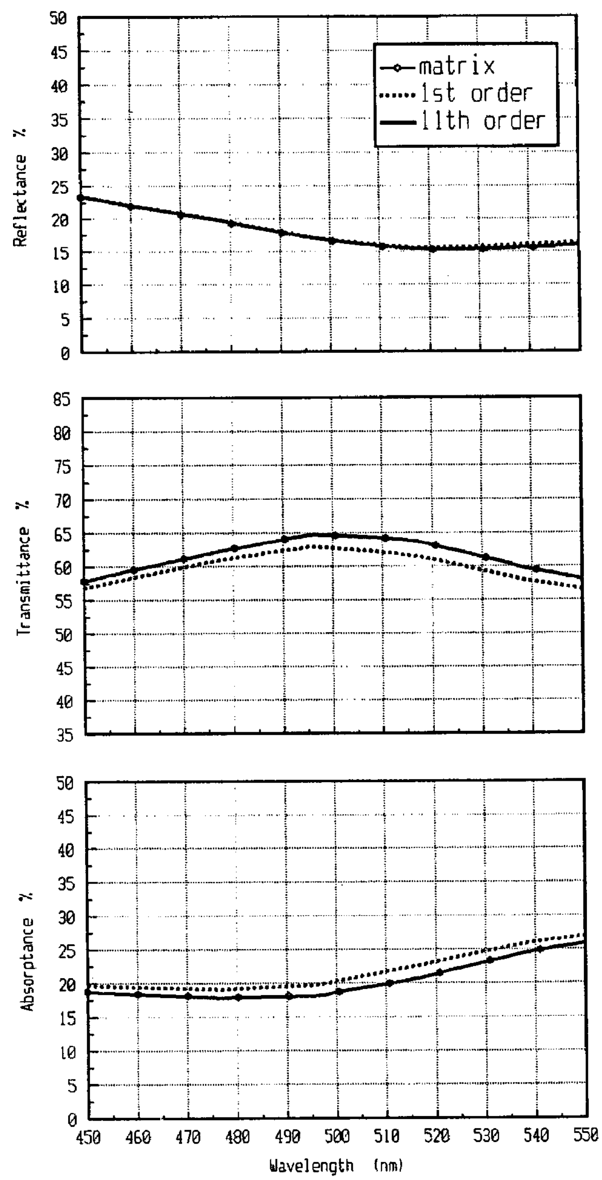

Fig. 2. Spectral performance of the beam splitter.

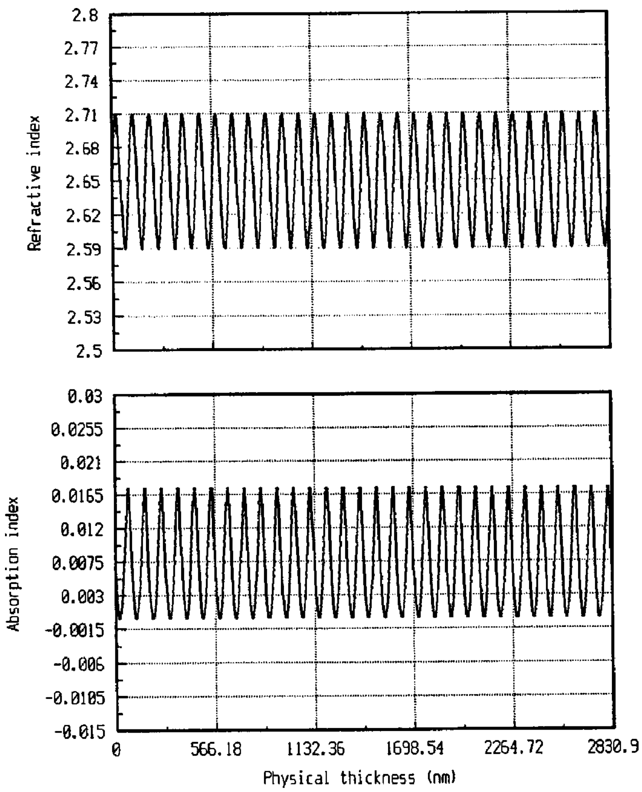

Fig. 3. Refractive- and absorption-index profiles of a rugate filter. The incident medium and substrate are also air and BK7, respectively.

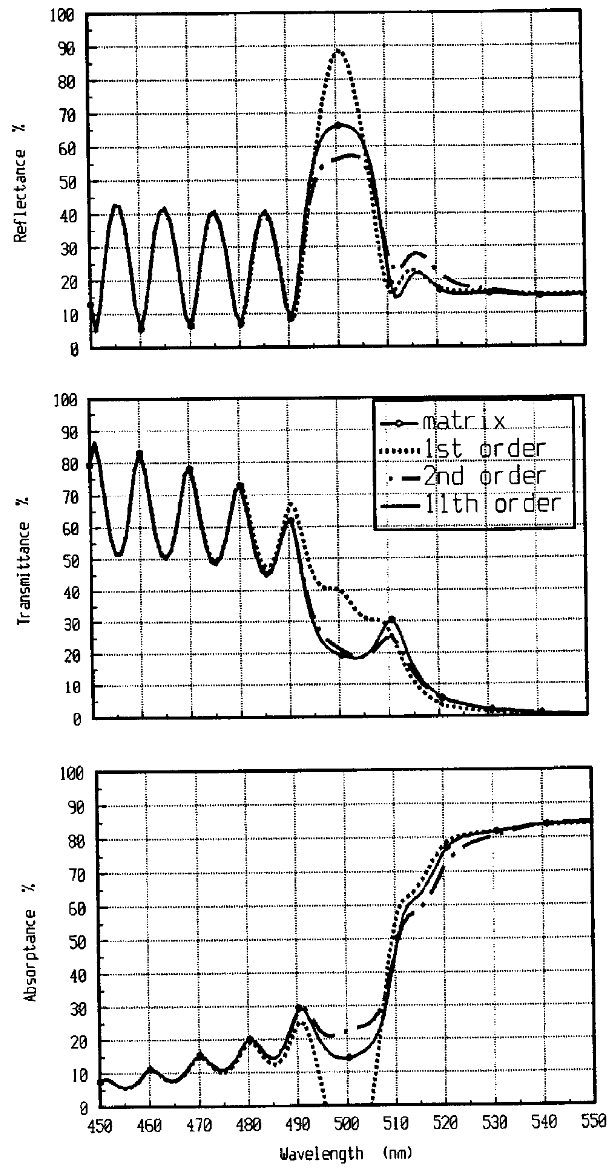

Fig. 4. Spectral performance of the rugate filter.

emerging media.[5] Then we have

$$R = \left|\frac{B_1 + B_3 + B_5 + \cdots}{B_0 + B_2 + B_4 + \cdots}\right|^2, \tag{8}$$

$$T = \frac{\mathrm{Re}(\eta_s)}{|\eta_s|}\,\frac{\exp i(\beta^* - \beta)}{|B_0 + B_2 + B_4 + \cdots|^2}, \tag{9}$$

$$A = 1 - T - R, \tag{10}$$

where

$$B_m = \begin{cases} \hat{C}_m(\beta) - i\hat{S}_m(\beta), & m = 1, 3, 5, \ldots, \\ \hat{C}_m(\beta) + i\hat{S}_m(\beta), & m = 0, 2, 4, \ldots. \end{cases} \tag{11}$$

The first term $B_0 = 1$. The argument in the exponential function in Eq. (9) is associated with the absorption process as the wave travels through the rugate cermet. In strongly absorbing thin films the exponential factor becomes dominant and the transmittance decreases rapidly with thickness as with pure metals.

The influence of the number of orders considered in Eqs. (8)–(10) is illustrated by two examples. The first is a beam splitter (Figs. 1 and 2). Here we can observe that the results from using the homogeneous matrix approach agree very well with those obtained from the inhomogeneous matrix, provided orders to the 11th order, B_{11}, are considered. In this case the first-order approximation differs by only $\sim$1%, which means that high reflected orders contribute weakly to the response.

In general dielectric inhomogeneous thin films make a strong contribution of higher-order terms to their spectral response in high-reflectance regions because of multiple reflections. However, in absorbing films high reflectance can be reached by increasing the absorption thickness $[\int_0^z k(z)\mathrm{d}z]$ with the consequent penalization of decreasing the influence of the inhomogeneity.

A particular example is a rugate absorbing filter (Figs. 3 and 4). In this case even the second-order approximation is really far from representing properly the exact response in the stopband region, in the same way as in the dielectric case. A design with a high-reflectance stopband is difficult to obtain when absorption is present, because the damping of the field through the film affects the interference responsible for the high-reflectance band.

Note here that the refractive and absorption indices cannot be defined arbitrarily because they must satisfy the Kramers–Kronig causality relations. To obtain the systems given in Figs. 1 and 3, we defined the refractive-index profile at a wavelength of 550 nm and then obtained the absorption index by using Bruggeman's effective medium model.[6] We used a mixture of gold and titanium dioxide, whose refractive and absorption indices were taken from Ref. 7 for the spectral region considered.

Following a procedure to obtain the inhomogeneous matrix given by Bovard, we derived the relations to calculate the reflectance transmittance and absorptance of rugate filters made of inhomogeneous thin films of cermets. The index of the mixture is obtained by the effective medium theory, and the resulting material satisfies closely the Kramers–Kronig relations.

Note that the homogeneous matrix method is computationally more efficient than the inhomogeneous method and gives comparable precision under the same resolution. However, the latter is physically more meaningful because it permits us to derive some properties of rugate filters. The examples were chosen to emphasize the main features of this method, which offers the possibility of performing synthesis by an analogous technique to that given for dielectric rugate filters. This is an interesting subject for future research.

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