lows from the first by putting $\epsilon = \epsilon' = 1$ (in more general case by the replacement $\epsilon = \mu$, $\epsilon' = \mu'$). Therefore, we need not write down Eqs. (2) and can get the values of Hz,s from those of Ez,s simply by putting $\epsilon = \epsilon' = 1$. From Eqs. (1) one obtains

$$E_z{}^r/E_z{}^i = (kz\,\epsilon' - kz'\,\epsilon)/(kz\,\epsilon' + kz'\,\epsilon). \qquad (3)$$

As noted above by setting $\epsilon' = \epsilon = 1$, we obtain

$$H_z{}^r/H_z{}^i = (kz - kz')/(kz + kz'). \qquad (4)$$

Because both $E_z{}^i$ and $E_z{}^r$ are in the same media, $|E_z{}^r/E_z{}^i|^2$ gives directly the reflectance for the p polarization. Similarly $|H_z{}^r/H_z{}^i|^2$ yields the reflectance for the s polarization. In fact the substitution $kz = k\cos\theta$, $kz' = k'\cos\theta'$, and $(\epsilon'/\epsilon)^{1/2} = k'/k = \sin\theta/\sin\theta'$, where θ and θ' are the angles of incidence and refraction, respectively, will give the familiar forms:

$$E_z{}^r/E_z{}^i = \frac{\tan(\theta - \theta')}{\tan(\theta + \theta')}$$

$$H_z{}^r/H_z{}^i = -\frac{\sin(\theta - \theta')}{\sin(\theta + \theta')}\,.$$

Next, we further apply the new method to the calculation of the transmission coefficient of light through a plate. The advantages of the method become increasingly clear as the complexity of the problem increases. As discussed above we need to write down only the boundary conditions which contain Ez,s

$$\epsilon(E_z{}^i + E_z{}^r) = \epsilon'(E_z{}^m + E_z{}^{m'}) \qquad (5a)$$

$$kz(E_z{}^i - E_z{}^r) = k'z(E_z{}^m - E_z{}^{m'}) \qquad (5b)$$

$$\epsilon E_z{}^t e^{ik_zd} = \epsilon'(E_z{}^m e^{ik'_zd} + E_z{}^{m'} e^{-ik'_zd}) \qquad (5a)'$$

$$kzE_z{}^t e^{ik_zd} = k'z(E_z{}^m e^{ik'_zd} - E_z{}^{m'} e^{-ik'_zd}), \qquad (5b)'$$

where $\mathbf{E}^m$ and $\mathbf{E}^{m'}$ are two waves inside the plate and d the thickness of the plate, one can easily obtain from these equations

$$E_z{}^t/E_z{}^i = \frac{4\,kz\,kz'\,\epsilon\,\epsilon'\,e^{-ik_zd}}{(kz\,\epsilon' + kz'\,\epsilon)^2 e^{-ik'_zd} - (kz\,\epsilon' - kz'\,\epsilon)^2 e^{ik'_zd}}. \qquad (6)$$

By setting $\epsilon = \epsilon' = 1$ we get

$$H_z{}^t/H_z{}^i = \frac{4\,kz\,kz'\,e^{-ik_zd}}{(kz + k'z)^2 e^{-ik'_zd} - (kz - k'z)^2 e^{ik'_zd}}. \qquad (7)$$

The absolute squares of Eqs. (6) and (7) yield the transmission coefficients of light through a plate for p and s polarizations, respectively. This simple and unified derivation preserves the simplicity of the problem under study and that of Maxwell's theory.

Finally, we discuss some other applications of this method, for example, transition radiation[2] emitted by an electron passing through the interface of two different media. One needs to know the values Ex and Ey in addition to Ez. However, the following equations give a very simple solution:

$$kxEx + kyEy = -kzEz \qquad (8a)$$

$$-kyEx + kxEy = (\omega/c)\,Hz \qquad (8b)$$

where Equation (8b) is the third equation of Faraday's law, and Equation (8a) follows from $\mathbf{k}\cdot\mathbf{E} = 0$. Therefore, we have an effortless procedure to find all field quantities: (1) Write down the conditions which involve Ez's only; (2) The values of Hz's follow from those of Ez,s by putting $\epsilon = \mu$, $\epsilon' = \mu'$; (3) Ex and Ey can be written down immediately in terms of Ez and Hz by Eqs. (8).

Especially in the problems of transition radiation one has to perform the inverse Fourier transforms, and the use of the transparent results obtained by the present method reduces a tremendous amount of work. For example, the tangential components of the propagation vector kx and ky are not contained in Ez and Hz and enter into Ex and Ey only through the simple transformation matrix given in Eqs. (8). Transparency attained by the new approach avoids unnecessary repetition of integrations for similar terms. Actually this work was prompted by the need to find a way out of the labyrinth of complicated algebra encountered in the theory of transition radiation. A detailed exposition of this and other applications of the present method will be given elsewhere.

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Does a Photon Have a Rest Mass?

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While it is generally stated that a photon has zero rest mass, it can nevertheless be shown on a formal basis that certain properties related to a finite rest mass are exhibited by photons in a material medium. This is in addition to and distinct from the mass of a photon associated with a gravitational field.[1]

Consider a beam of photons in vacuum having a frequency ν. Let the intensity of the beam be given by

$$S = Nh\nu c \qquad (1)$$

where S, the Poynting vector, is the energy crossing unit area in unit time; $Nh\nu$ is the energy density, and N is the number of photons per unit volume. The quantities h and c are Planck's constant and the velocity of light in vacuum, respectively.

Let us assume that the beam of photons impinges on a medium whose index of refraction is n and penetrates without loss of energy; i.e., the medium is nonabsorbing and nonreflecting. The latter assumption is really not necessary but is given for convenience. One could, for example, either take into account the small amount of reflected energy or else remove this problem by interposing a layer which reduces the reflection to zero by interference.

The conservation of energy then gives that $S =$ constant or

$$Nh\nu c = N'h\nu v, \tag{2}$$

where N' is the photon density in the medium, and v is the light velocity in the medium given by $v = c/n$.

The momentum density of the electromagnetic field in vacuum is given by $G = Nh\nu/c$ and as a consequence it is obvious that the momentum density is related to the Poynting vector by

$$G = S/c^2. \tag{3}$$

The relation $G = S/c^2$ can be taken as a consequence of the symmetry of the electromagnetic energy-momentum tensor.[2]

The relevancy of Eq. (3) to material media has been a subject of past inquiry. More recently the symmetry requirement on the energy-momentum tensor has been discussed and questioned by Møller.[3]

If we use the classical[4] assumption that the energy-momentum tensor is to be symmetric in material media as well as in vacuum, then an application of Eq. (3) to the material medium gives

$$N'p' = (1/c^2)\, Nh\nu c, \tag{4}$$

where p' is the photon momentum in the medium.

From Eq. (2) we have the photon density in the material medium as $N' = nN$. Combining this result with Eq. (4) we arrive at the fact that the photon momentum in the medium is less than it is in vacuum

$$p' = (1/n)\,(h\nu/c). \tag{5}$$

Let us now approach the problem from an entirely different direction.

If we formally consider the photon to act like a particle moving with a velocity v and having an energy, E, we may, using the standard relativistic equation, write its momentum as

$$p' = (E/c^2)\, v. \tag{6}$$

From this one obtains immediately

$$p' = (h\nu/c^2)v = (h\nu/c)(1/n),$$

which is identical with Eq. (5). In other words one may alternatively derive Eq. (5) by treating the photon in the medium like a material particle whose mass is m_{ph} and whose momentum is

$$p' = \frac{m_{ph}\, v}{(1 - v^2/c^2)^{1/2}},$$

that is, the effective photon mass is given by

$$m_{ph} = (h\nu/c^2)\,(1 - n^{-2})^{1/2}. \tag{7}$$

The value of m_{ph} is, as expected, exceedingly small. If we choose the wavelength as $\lambda = 0.5\ \mu$ and let the index of refraction be $n = 1.3$, the photon mass is $m_{ph} = 2.82 \times 10^{-33}$ gm or $m_{ph} = 3.23 \times 10^{-6}\, m_e$, where m_e is the electron mass. The photon mass increases with frequency so that at x-ray frequencies one picks up a factor of perhaps 10^3. However, remembering that the index of refraction is practically unity for this frequency, one finds that an x-ray photon also has, in practice, an exceedingly small mass.

What is the significance of the photon mass (small or not)? It does not seem unreasonable to believe that in the event of a "real" photon collision the photon would exhibit its zero rest mass in spite of the formal nonzero mass. This is probably analogous to the fact that even though a conduction electron in a metal may have the properties of a particle whose effective mass is different from its real mass (because of the effect of interaction with the crystal lattice[5]) the "real" electron mass manifests itself in inertial experiments.

Finally one may ask whether there exists any circumstance in which the photon mass could be "measurable." A way of looking at this is to compare photon mass density with energy density. We see that for the 5000-Å photons an energy density of 10 eV per 10^{-23} cm^3 (approximately an atom or molecule volume) would give rise only to about 1.4×10^{-5} electron masses and that such an energy density would be more than sufficient (about 15 times the heat of vaporization of water) to evaporate most materials.

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[4] J. A. Stratton, *Electromagnetic Theory* (McGraw Hill Book Co., New York, 1941), p. 158 quoting the earlier work of M. Abraham and M. Von Laue.

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